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The graph coloring problem and NP-Completeness

The graph coloring problem, also known as the map coloring problem, is deceptively simple. In the context of maps, the problem is to color every separate region on the map, without any two areas which share a border receiving the same color, and do so using the minimum possible number of colors [1]. Although the coloring of maps is not as critical of a task as it once was, this problem nonetheless has many applications in the modern world. The graph coloring problem can be used to model and find solutions for scheduling problems, both in general and in more specialized applications, such as scheduling tasks for CPU registers [3][4].

As mentioned previously, the graph coloring problem is deceptively simple, the problem itself is not particularly complex, however the solutions to it are anything but simple, or fast [2]. In fact, the graph coloring problem is what's known as an NP-Complete problem [4]. In NP-Complete, NP stands for Non-Polynomial deterministic time, and Complete represents the class of NP problem. Non-Polynomial deterministic time (NP) may sound utterly nonsensical, however its meaning is quite important.

An NP problem is a problem for which there exists no known algorithm for finding a solution that has a run time of less than or equal to polynomial time for a deterministic machine. Conceptually, these kinds of problems can be thought of as problems for which a brute-force approach, where every possible solution is generated and checked, may in fact be the only known method for reliably finding an exact optimal solution. NP-Complete is an extension of this, it retains the extreme difficulty in computation of an NP problem, however with one important extra condition. This extra condition requires that there is a known algorithm which can determine if a given solution is in fact correct, and do so in at most polynomial time.

Although providing an exhaustive proof that the graph coloring problem is an NP problem is both beyond the scope of this paper and beyond the abilities of its writer, the graph coloring problem has been known to be an NP-Complete problem for nearly a half century [2]. However, the previously mentioned extra condition for an NP-Complete problem, that it requires a polynomial or faster method to verify the solution, can be easily explained for this problem. To find which solution produced by a brute force approach is the optimal solution for this problem, the test can simply loop over all the provided solutions and pick the one with the smallest value.

Given the extreme time complexity of these problems and their utility in everyday life, it comes as no surprise that significant resources and time have been spent in attempts to find efficient solutions to problems such as the graph coloring problem. Efforts such as those by Mahmoudi et al [3], and Dokeroglu et al [4], have opted to sacrifice the correctness of the algorithm for a dramatic increase in speed. For a number of applications, this dramatic increase in speed is well worth the loss in accuracy, but accuracy is still a very desirable quality. Work continues on this front to improve the speed of known algorithms for solving the graph coloring problem.

References

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